Theory and Modeling of Internal Wave Generation in Straits

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LONG-TERM GOALS

The long-term goal is to improve understanding of the generation of nonlinear internal tides and waves by stratified flow over sills (e.g. straits) and shelves and the subsequent evolution of the radiated internal disturbance, with particular emphasis on the role of rotation.

OBJECTIVES

The primary objective will be on connecting the generation process with the dynamics of the disintegration of the radiated internal tide into shorter, nonlinear internal solitary-like waves and to predict the space and time scales for the emergence of waves and their properties (e.g. wave amplitudes, numbers, etc.). A central aspect of this work is to explore the role of rotation in the process. Rotation permits the presence of periodic, nonlinear inertia-gravity waves (i.e., the tide) that can act as attractors and arrest the steepening of the internal tide, and hence affect the production of the shorter solitary-like waves (Gerkema, 1996; Helfrich and Grimshaw, 2008). A related objective is the long-term effect of rotation on the emerging solitary waves, where it has been shown that these waves may decay to procude nonlinear wave packets (Helfrich, 1997; Grimshaw and Helfrich, 2008). Exploration of the role of propagation in two horizontal on these processes is also a goal.

APPROACH

The approach combines theoretical wave evolution models and numerical solutions of these models and solutions of the full Navier-Stokes equations. The theoretical models require some simplifications that, depending on the specific situation, may include restriction to two-layer flows, one-dimensional propagation, weak nonlinearity and either weakly non-hydrostatic or fully hydrostatic dynamics. The models used include those in the Korteweg-de Vries (KdV) family of equations modified to include higher-order nonlinearity, rotation, and transverse propagation. Fully nonlinear dynamics can be explored through an extension of the fully nonlinear, weakly non-hydrostatic internal wave theory of Miyata (1988) and Choi and Camassa (1999) to include rotation (Helfrich, 2007). In order to study the generation process, variable topography has been included in the model. These reduced wave equation models are complemented using 2.5- and 3-dimensional Navier-Stokes numerical models when appropriate. The use of reduced models is a drastic simplification in many situations, but the insight obtained from these models can guide the analysis of more complex models and observational data

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obtained as part of the IWISE field study. Laboratory experiments on the LEGI-Grenoble 13 meter diameter rotating platform have also been conducted.

WORK COMPLETED

Work during the last 12 months has focussed primarily on the steepening dynamics of the nonlinear internal tide and the evolution of internal solitary waves in the presence of rotation. The rotational decay of a single internal solitary wave into a nonlinear wave packet was explored in a set of laboratory experiments and comparison with theory and models (Grimshaw and Helfrich, 2012; Grimshaw, Helfrich, and Johnson, submitted). A criterion for the breaking of the internal tide, and the subsequent production of internal solitary-like waves, has been derived theoretically and further explored to demonstrate the significance and utility of the criterion in predicting the occurance of breaking (Grimshaw, Helfrich and Johnson, 2012). This criterion is being applied to observational data taken in the South China Sea during 2007. The effect of transverse structure of the internal tide on the sttpening and breaking process has been explored.

RESULTS

Is has been know for some time that rotation leads to the deacy of an initial solitary waves through a direct resonance with Poincare waves (Grimshaw, Ostrovsky and He, 1998; Melville, Tomasson and Renouard, 1989). Recent numerical and theoretical work has shown that the long-time result of this radiational decay is a nonlinear internal wave packet governed by an extended nonlinear Schrodinger equation (Helfrich, 2007; Grimshaw and Helfrich, 2008). In September 2009 a series of laboratory experiments were performed at the LEGI-Coriolis 13-meter rotating platform in Grenoble, France to test these ideas. The experiments were conducted using a two-layer stratification with interfacial waves generated by a lock-exchange from a lock that was 5 meters long in the direction transverse to the wave propagation direction. The lock height and the table rotation rate were varied during the experiments. Figure 1 shows a summary of the experimental results for a interfacial difference across the lock of *9* cm. The interfacial displacement, or wave amplitude η, recorded by interfacial displacements probes at 1 m and 7.64 m downstream from the lock are shown. Without rotation (Coriolis parameter $f = \infty$) the lead disturbance is a well-defined solitary wave with constant propagation speed. The introduction of rotation leads to the emergence of a leading wave packet. The localized character of the packet is an indicator of nonlinear wave packet dynamics. Figure 2 shows an eample of a simulation of one of the experimental runs using a 3D non-hydrostatic numerical model. The packet structure emerges despite the geometric spreading inherent in the experimental set up. The simulation is in good agreement with the experiments (see Figures 2 c and d) The experiments also show that more general initial conditions (e.g., geostrophic adjustment) will also produce the wave packets. This work is important since it broadens the type of wave groups that can be expected and may provide an explanation for observations that do not fit in the classic paradigm of a rank-ordered group of solitary-like waves. These results are presented in an article reviewing the theoretical analysis and some preliminary experimental data (Grimshaw and Helfrich, 2012). The complete experimental results have been submitted for publication (Grimshaw, Helfrich, and Johnson, submitted).

In many situations such as the South China Sea, stratified tidal flow over localized topography leads to the radiation of a low-mode internal tide. This internal tide many subsequently steepen due to nonlinearity and produce internal solitary-like waves. However, it is know that rotation acts to slow the steepening and may prevent the emergence of the shorter solitary-like waves (Gerkema, 1996; Helfrich and Grinshaw, 2008). The question of whether an internal tide will steepen has been addressed using the Ostrovsky equation (the Korteweg-de Vries equation extended to include)

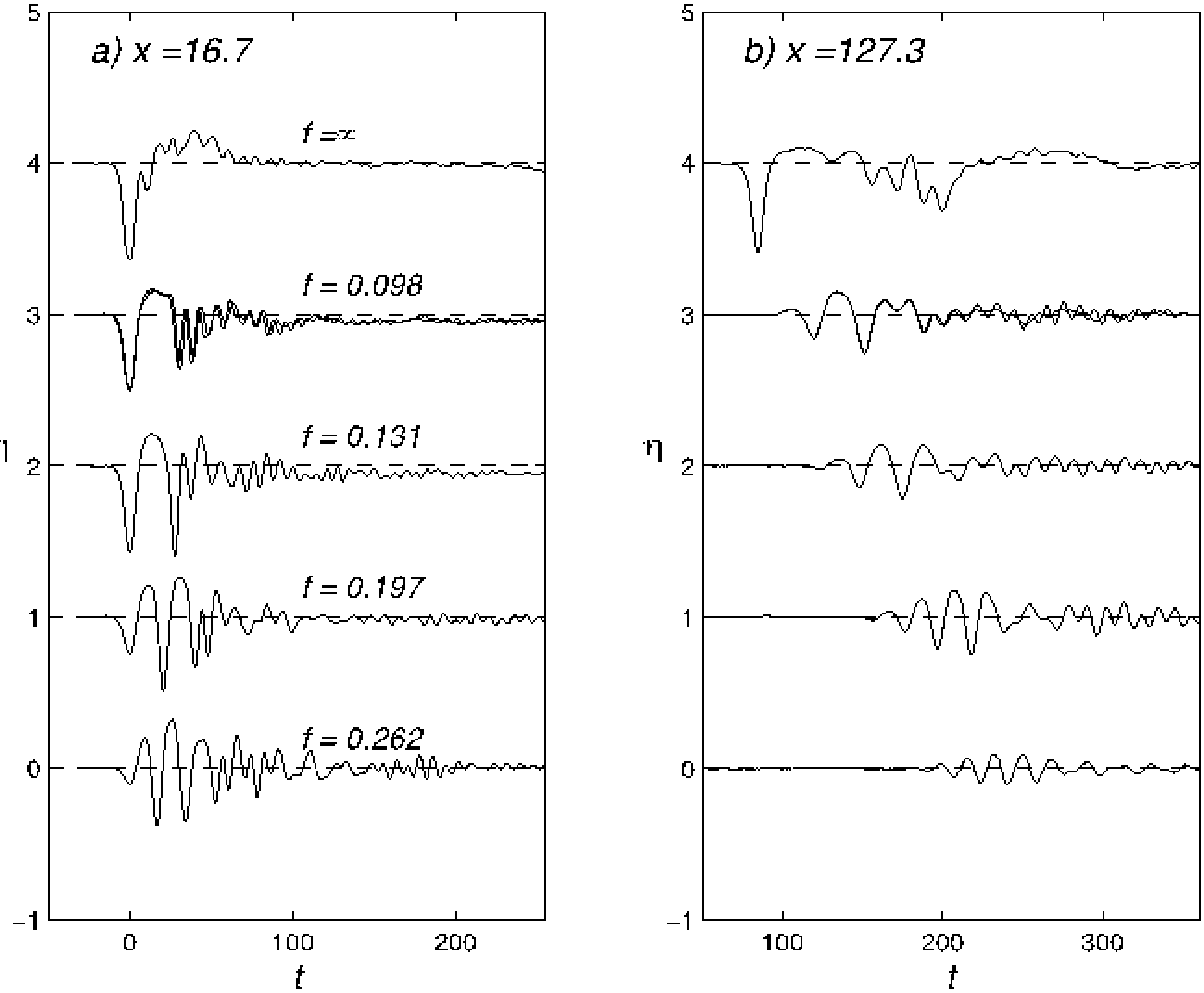

Figure 1. Wave amplitude η along the centerline of the experiment (y = 0, see Figure 2) as a function of non-dimensional time t for runs with a 9 cm lock height with non-dimensional Coriolis frequency f as indicated. a) The wave amplitude at non-dimensional x = 16.7 (= 1 m) from the lock. b) At x = 127.3 (= 7.64 m).

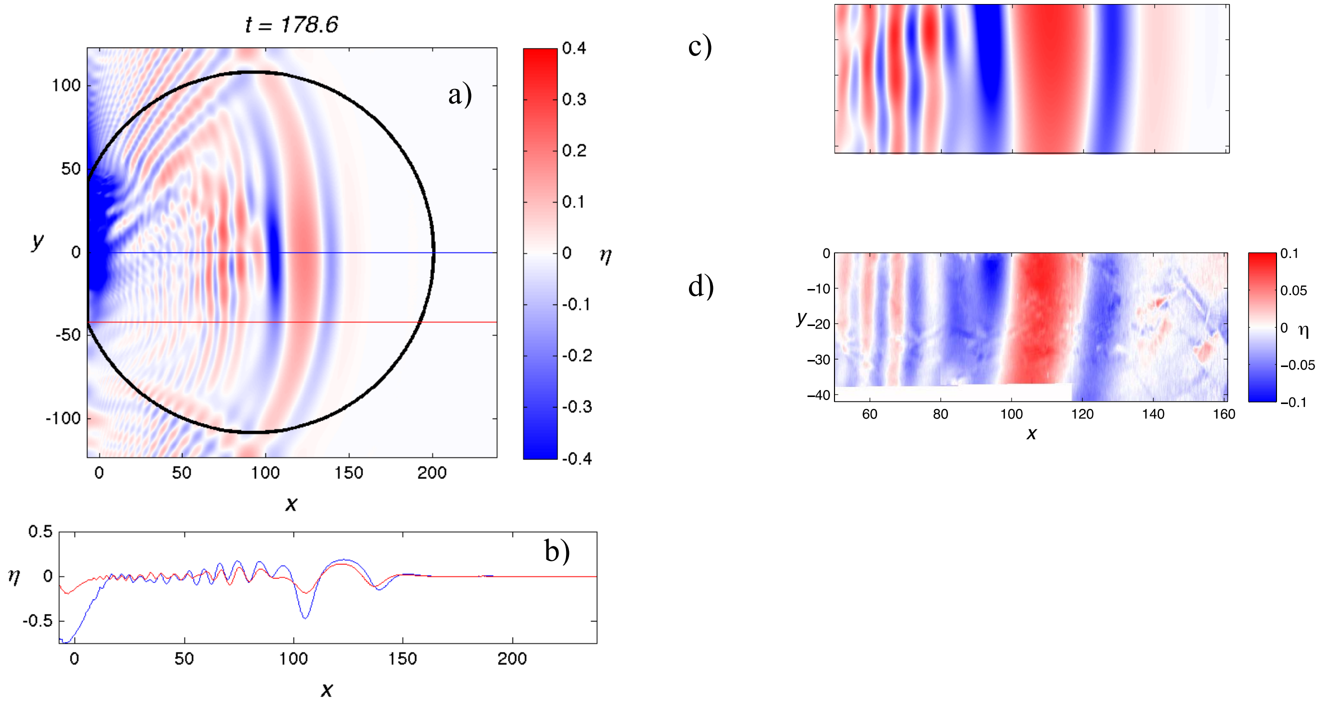

Figure 2. a) Wave amplitude η(x,y,t) from a 3D non-hydrostatic simulation of an experimental run with table rotation period of T = 90 s and a 9 cm lock depth. The black circle indicates the position of the experimental tank boundaries. b) The numerical solutions along the centerline (blue) and at the edge of the wave generator (red) as indicated in (a). c) A close-up of the 3D simulation at t = 181. d) The wave amplitude from an experiment at t = 181 with the same parameters as (a)-(c).

$$\frac{\partial}{\partial x}\left(\frac{\partial \eta}{\partial t}+\alpha\eta\frac{\partial \eta}{\partial x}+\beta\frac{\partial^3 \eta}{\partial x^3}\right)=\gamma\eta \qquad (1)$$

for the wave amplitude $\eta(x,t)$. The coefficients of the nonlinear, α, nonhydrostatic, β, and rotational terms, γ, are functions of the background stratification. In the hydrostatic limit ($\beta = 0$) the reduced-Ostrovsky equation has been used as a model for the nonlinear evolution of the internal tide. Boyd (2005) has shown from numerical calculations that the sinusoidal initial condition $\eta = a \sin(kx)$ will evolve to breaking if $3\alpha a k^2/\gamma > 1$. Farmer, Li and Park (2009) and Li and Farmer (2011) have used Boyd's breaking criterion to predict the conditions which lead to emergence of internal solitary-like waves in the South China Sea.

However, the general application of this criterion is limited by the fact that it is essentially an empirical criterion applicable only to sinusoidal initial conditions. Recently we have shown theoretically that breaking will occur in the reduced-Ostrovsky equation for *any* initial condition that possesses a region where the Ostrovsky number $O_s = 3\alpha\eta_{xx}/\gamma > 1$ (Grimshaw, Helfrich, and Johnson, 2012). For the siunsoidal initial condition this reduces to Boyd's criterion. However, this new criterion is a more powerful statement since it is a local test applied at every x of the initial condition. Also it is a condition on the maximum curvature of the initial condition, and not the slope as Farmer, Li and Park (2009) and Li and Farmer (2011) have interpreted Boyd's criterion. Numerical tests with general initial conditions demonstrate that the number of independent breaking events is equal to the number of distinct regions in the initial condition that have $O_s = 3\alpha\eta_{xx}/\gamma > 1$.

The significance of this new result for oceanographic conditions is being assessed by application of the criteria to the 2007 South China Sea PIES observations of Li and Farmer (2011). (Data kindly supplied by David Farmer.) Figure 3 shows an example of the analysis. The panels in the left column show the wave amplitude η derived from the PIES instruments at the three moorings A1, A2, and A3 that run along an east-west line from between the two ridges in the Luzon Strait (A1) to about the 2500 m isobath (A3) (see Li and Farmer (2011) for mooring location details). The black lines are the amplitudes filtered to remove noise inherent in the PIES data, but leaving the internal tide and solitary-like waves. The red lines are the amplitude data filtered to remove the solitary-like waves, leaving only the internal tide, η_{tide}. Under some conditions solitary waves emerge from the internal tide by moorings A2 and A3. The top panel of the right column of Figure 3 shows the Ostrovsky number computed using the (red) internal tide signal at mooring A1. The continuous stratification of the SCS during the mooring period was used to compute the coefficients of (1). The bottom two panels show the amplitude of the solitary-like waves found by subtracting the filtered internal tide signal from the complete signal $\eta_{ISW} = \eta - \eta_{tide}$. The emergence of solitary-like waves at A2 and, especially, A3 is correlated with periods of $O_s > 1$. This analysis is ongoing, but these results are encouraging and indicate the utility of the Ostrovsky number breaking criterion.

Recent observations in the South China Sea have focussed on the two-dimensional (x-y) structure and evolution of the internal tide and solitary waves. For example, how does the transverse structure of the internal tide radiating west from the Luzon Strait affect the steepening process described above? This issue is being explored using the rotating Kadomstev-Petviasvilli (KPf) equation

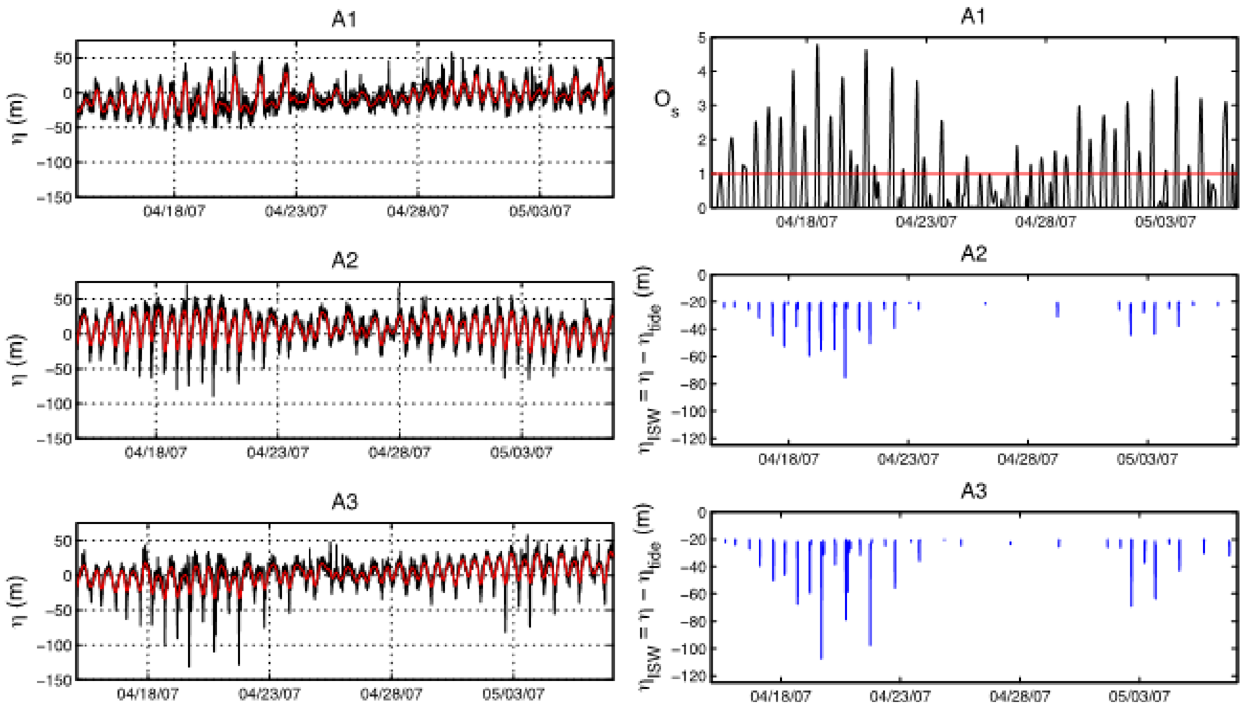

Figure 3. Left column: A portion of the wave amplitude η from the 2007 PIES data of Li and Farmer (2011) at the A1, A2 and A3 moorings as indicated. The black line shows η filtered to remove the high-frequency noise inherent in the PIES observations, leaving the internal tide and solitary-like waves. The red lines show η filtered to leave the low-frequency internal tide only. The signals at A2 and A3 have been shifted by the propagation times from A1 to these moorings (see Li and Farmer, 2011). Right panel: The top panel shows the Ostrovsky number (black lines) calculated from the filtered internal tide signal at A1. The red line shows the critical value for breaking. The lower panels show the internal solitary-like wave signal at moorings A2 and A3 found by subtracting the internal tide from the full signal $\eta_{ISW} = \eta - \eta_{tide}$ (black minus red in the left panels). Only signals with $\eta_{ISW} < -20$ m are shown.

$$\frac{\partial}{\partial x}\left(\frac{\partial \eta}{\partial t}+\alpha\eta\frac{\partial \eta}{\partial x}+\beta\frac{\partial^3 \eta}{\partial x^3}\right)=\gamma\eta-\sigma\frac{\partial^2 \eta}{\partial y^2} \qquad (2)$$

which is an extension of the Ostrovsky equation for weak transverse (y) effects. Figure 4 shows example numerical solutions of (2) for an initial condition that is sinusoidal in x with a Gaussian shape in y with width scale W. The calculation uses coefficients of (2) calculated for a typical South China Sea stratification. The length in x of the initial sinusoidal wave corresponds the M2 internal tide. The initial (scaled) amplitude $a = 0.06$ is greater than the critical amplitude breaking amplitude $a_c = 0.025$ for $O_s = 1$ in the reduced-Ostrovsky equation. With no transverse effects ($W = \infty$) this initial wave leads to the emergence of a packet of solitary waves as expected. However, the production of solitary waves is inhibited as W is decreased until for $W = 1$ there are no waves at the time shown in the figures (about 2.3 days in the SCS). This work is being finalized for publication.

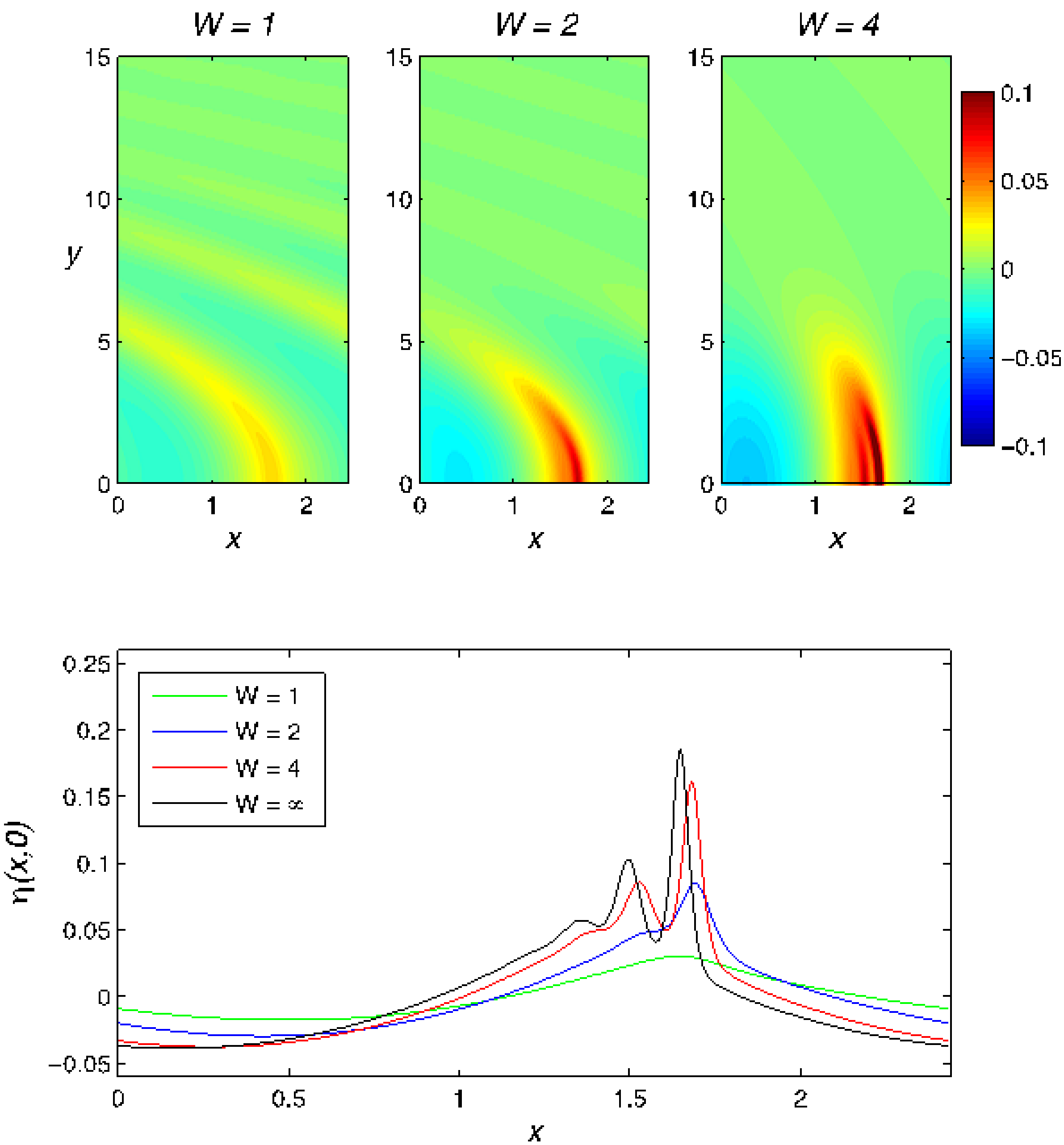

Figure 4. Numerical solutions of the rotating KP equation (2) for an initial sinusoidal (in x) internal tide. The initial condition has a Gaussian shape in the transverse y direction with width scale $W = 1$, 2, 4, and ∞ as indicated. All solutions have the same initial amplitude, equation coefficients, and are shown at the same time. The upper panels show the plan view structure of the wave amplitude η for W as indicated. The solutions are symmetric about $y = 0$ so only $y \geq 0$ is shown. The bottom panel shows η along the mid-line $y = 0$.

IMPACT/APPLICATIONS

The ubiquitous nature of large amplitude internal solitary waves in the world's coastal oceans and marginal seas is clear from observations. These waves are can have significant effects on coastal mixing through breaking as they propagate and shoal, and they may also lead to substantial horizontal mass transport. Since the waves are frequently generated through the radiation of an internal tide by barotropic tidal flow over localized topography (as is apparently the case at in the Luzon Strait), this work will help understand what fraction of the energy put in at the tidal frequency ends up as internal solitary waves, the space and time scales for that transformation, and the characteristics of the resulting solitary-like waves.

RELATED PROJECTS

This project is part of the IWISE DRI. It is also closely related to the PI's collaboration on the ONR MURI on Integrated Oceanographic, Atmospheric, and Acoustic Physics titled: "Integrated Modeling and Analysis of Physical Oceanographic and Acoustic." In that project, the PI is responsible for efforts to integrate reduced-dynamics wave models into regional hydrostatic coastal circulation models for the estimation of internal wave conditions and their effect on acoustic propagation. Some of the basic wave dynamics work funded by this grant is directly applicable to the MURI effort.

REFERENCES

Boyd, J., 2005. Microbreaking and polycnoidal waves in the Ostrovsky-Hunter equation. *Phys. Lett. A* **338**, 36-43.

Choi W. and Camassa, R. 1999. Fully nonlinear internal waves in a two-fluid system. *J. Fluid Mech.* **396**, 1-36.

Farmer, D., Q. Li, and J-H Park, 2009. Internal wave observations in the South China Sea: The role of rotation and nonlinearity. *Atmos.-Ocean* **47**, 267-280.

Gerkema, T., 1996. A unified model for the generation and fission of internal tides in a rotating ocean. *J. Mar. Res.* **54**, 421-450

Grimshaw, R., J.-M. He, and L. Ostrovsky, 1998. Terminal damping of a solitary wave due to rotation in rotational systems. *Stud. Appl. Math.* **101**, 197-210.

Helfrich, K. R., 2007. The decay and return of rotating internal solitary waves. *Phys. Fluids* **19**, 026601.

Helfrich, K. R. and R. H. J. Grimshaw, 2008. Nonlinear disintegration of the internal tide. *J. Phys. Oceano.* **38**, 686-701.

Li, Q. and D. Farmer, 2011. The generation and evolution of nonlinear internal waves in the deep basin This of the South China Sea. *J. Phys. Oceano.* **41**, 1345-1363.

Miyata M. 1988. Long internal waves of large amplitude. In *Nonlinear Water Waves*, IUTAM Symposium Tokyo 1987, ed. K. Horikawa and H. Maruo, pp. 399-406, Berlin:Springer Verlag.

PUBLICATIONS

Grimshaw, R. and K. Helfrich, 2012. Effect of rotation on internal solitary waves. *IMA J. Appl. Maths.*, doi: 10.1093/imamat/hxs024.

Grimshaw, R., K. Helfrich, and E. R. Johnson, 2012. The reduced-Ostrovsky equation: Integrability and breaking. *Studies Appl. Math.* **129**, 414-436.

Grimshaw, R., K. Helfrich, and E. R. Johnson, Experimental study of the effect of rotation on large amplitude internal solitary waves. *J. Fluid Mech.*, submitted.